**Adaptive spatial discretizations coarsen where acceptable accuracy can be achieved on coarse grids.**

**Heterogeneous media requires high resolution throughout the domain.**

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### NONLINEAR SOLVERS

The prevailing nonlinear solution algorithms are based on global linearization, using either Newton or Picard iteration.

\[ F(u) = 0 \]

\[ \text{Solve: } J(u)v = -F(u), \quad u \leftarrow u + v \]

\[ \gamma(u) = \frac{1}{2} \nabla u^2 \]

- Each iteration requires a global linear solve (e.g., Krylov-Multigrid).
- Each iteration moves important information over large distances.
- Superlinear convergence not realized for nonsmooth problems.
- The number of iterations depends on the strength of the nonlinearity.

### MODEL PROBLEM: p-LAPLACIAN WITH FRICTION

- 2-dimensional model problem for power-law fluid cross-section, \( 1 \leq p < \infty \)
- \( \nabla \cdot (\gamma \nabla u) - f = 0 \)
- Friction boundary condition, \( 0 < q \leq 1 \)
- \( \nabla u \cdot n + A(x)|u|^{p-1}u = 0 \)

### CONVERGENCE OF RESIDUALS

- Adaptive solutions show good convergence.
- Convergence is robust to strong, long-range effects.

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### REFERENCES