Higher Standards on the Control of Numerical Accuracy **2015-12-01**

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Abstract

The relatively low current standard for control of numerical accuracy is hurting climate models in many ways. It is crucial for reliability, trust, and the efficient use of scientist time that we become more systematic about verification—quantifying the extent to which we solve the stated continuum equations, especially as climate models transform into engineering tools that advise policy and investment decisions.

Background/Research to Date

The 1986 Editorial Policy Statement for the Journal of Fluids Engineering [5] states

> A professional problem exists in the computational fluid dynamics community and also in the broader area of computational physics. Namely, there is a need for higher standards on the control of numerical accuracy. [...] It [is] impossible to evaluate and compare the accuracy of different turbulence models, since one [cannot] distinguish physical modeling errors from numerical errors related to the algorithm and grid. This is especially the case for first-order accurate methods and hybrid methods.

The statement goes on to mandate systematic evaluation of numerical errors. This policy has been influential in defining the standards of the computational engineering community, for whom validation is readily available because products are manufactured and tested. Models that do not ascribe to this standard are widely viewed as less reliable and lower quality.

Climate models are being sold to policy makers as engineering tools, but with much lower verification standards and with no comparable validation process (due to the time scales involved and practical infeasibility of avoiding overfitting due to ad-hoc calibration process). Indeed, many prominent models have grave deficiencies.

- MPAS [7] uses an inconsistent discretization of Coriolis on non-orthogonal grids [6], including at the pentagons that are necessary when tiling the sphere with hexagons.
- CICE relies on the EVP formulation which introduces artificial parameters that materially affect solutions [3, 4] and efficiency.
- CAM column physics uses a splitting scheme that is not temporally convergent and significantly affects computed solutions [2, 1].

Models are calibrated to compensate for the systematic numerical errors produced by such schemes, thus making it difficult to distinguish the properties of the continuum model from that of the discretization. More concerning is that experimentation with improved discretizations requires expensive, ad-hoc recalibration involving significant domain expertise.

Proposed Direction of work

New and existing models must be reevaluated to distinguish modeling error from numerical error. Numerical errors should be controlled by improving discretizations, addressing deficiencies in decreasing order of prominence. In cases were it is computationally infeasible to control numerical error to a similar scale as modeling error, the parameter calibration procedure must be formalized such that it can be rapidly tuned for a new discretization. A more formal calibration process should be encouraged throughout the model, but it is especially important in such cases, for scientific understanding, to establish confidence, and to encourage improvements. Note that it is unacceptable to evaluate numerical methods in idealized scenarios with artificial viscosity (e.g., hyperviscosity) turned off, then use it with such terms present.

Specific solutions to fix non-convergent methods while maintaining efficiency will likely involve increased use of implicit methods and less reliance on naive splitting methods. While possibly more technical to develop, these techniques should improve algorithmic scalability and better expose fine-grained parallelism.

Connections to Math, Comp Sci & and Climate Science

The first goal for a mathematician working on numerical methods is usually verification, demonstrating that a particular algorithm solves the given continuum equations to some accuracy. This concept is relatively easy to explain and is accessible even to those who lack domain expertise. If the concept of convergence and metrics for evaluation is made prominent, it creates an environment in which domain scientists can effectively communicate with applied mathematicians and computer scientists and spur research likely to have a direct impact on the field. In the current system, it is far too common for researchers to spend time on methods that will never be viable for poorly specified reasons, resulting in dead-end abstract work and/or moving on to research in other disciplines. A more systematic approach to discretization errors will likely encourage refactoring models to more precisely define the equations, thus improving software modularity and maintainability and making it easier to utilize general-purpose numerical libraries.

Potential Impact on the field

A systematic approach to numerical accuracy should lead to adoption of convergent numerical methods yielding higher quality predictions and reducing the effort required to recalibrate a model when the discretization is changed. It will aid communication and make research on improved numerical methods more appealing.

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